Math Virtual Learning

Precalculus
Period Changes to Trig Graphs

April 20, 2020

Precalculus
Lesson: April 20th, 2020

Objective/Learning Target:
Students will be able to determine the period of a trigonometric function. The student will also be able to adjust a graph of a trigonometric function based on any changes to the period.

Let's Get Started:

What is the period of a trigonometric function?

Watch the video below to see an explanation of the period of a trig function.

Please note that he misspeaks at 5:34. He meant to say pi/2.

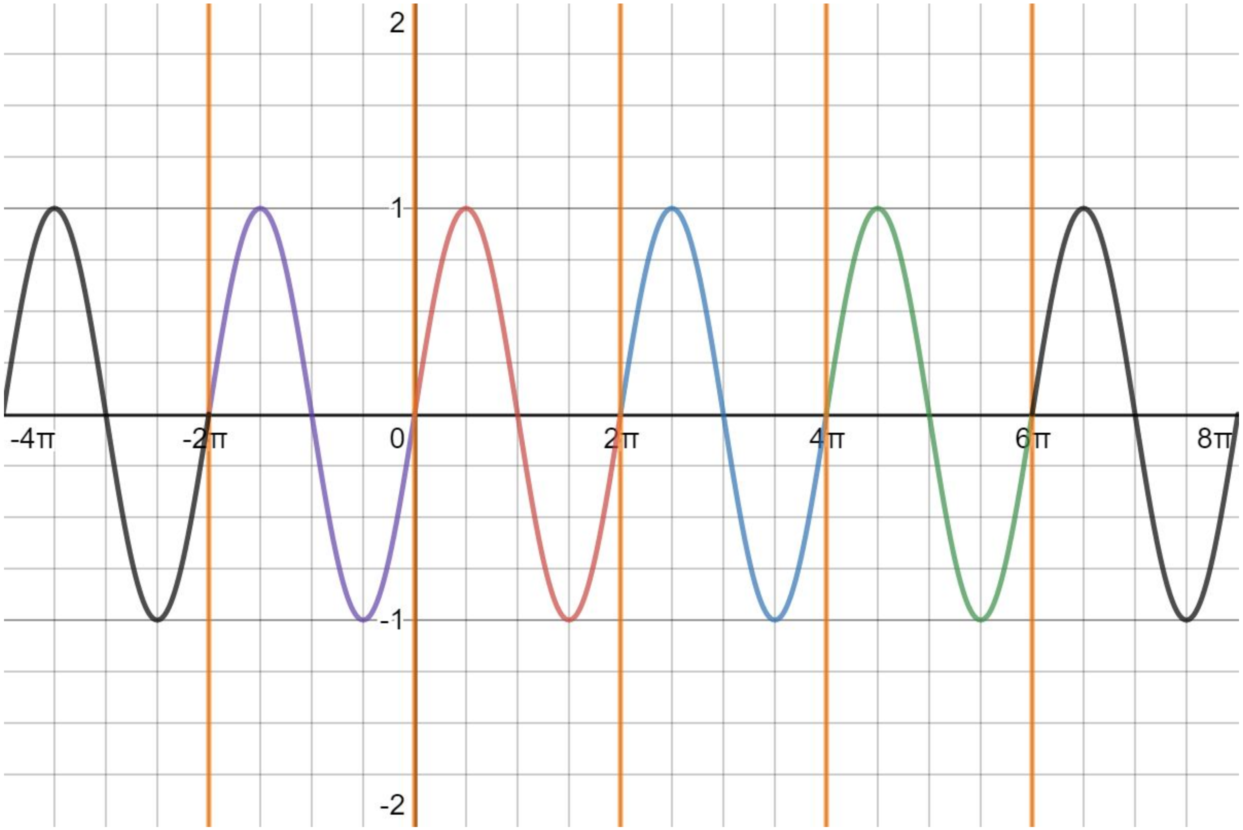
Video: How do we find the period of our trigonometric graphs sine and cosine?

Period of a Trigonometric Function

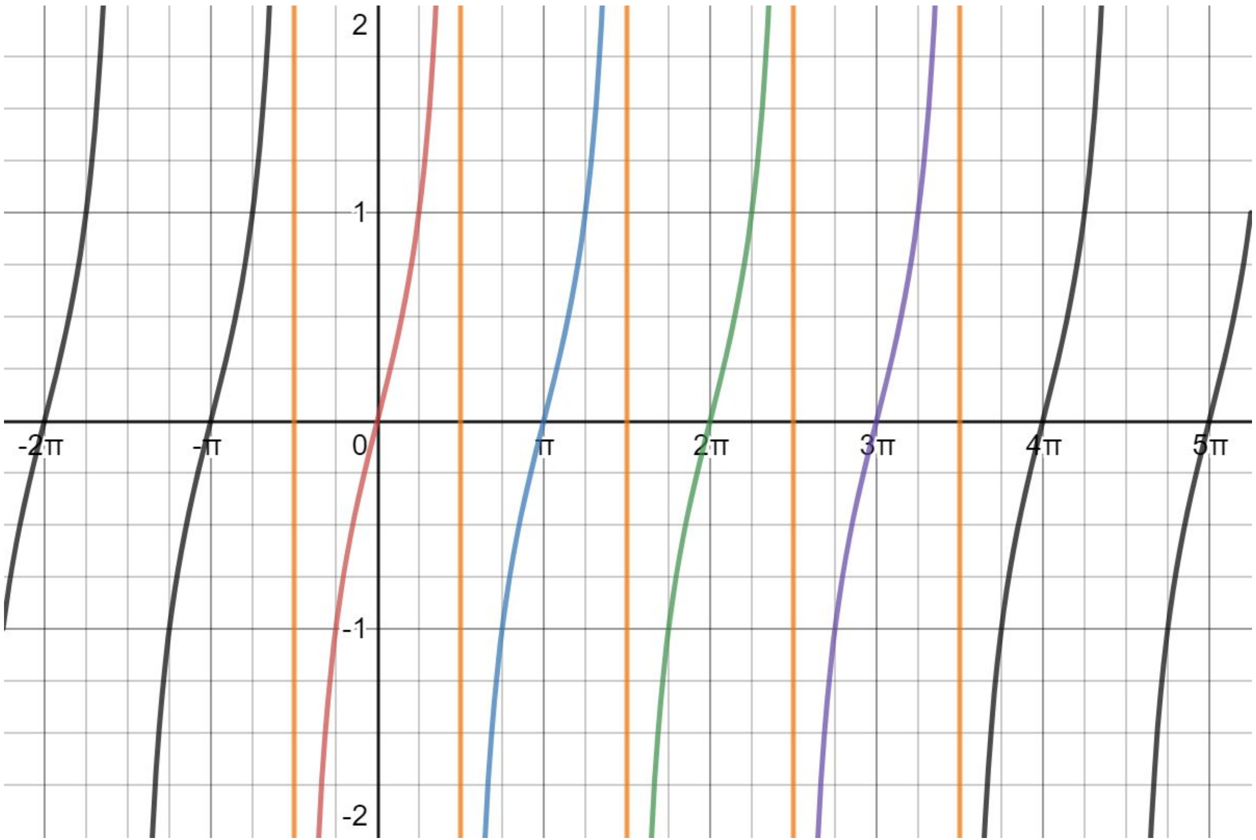
So the Period of a trigonometric function is the distance (usually along the x-axis) that it takes a trig function before it starts to repeat itself.

	PERIOD OF PARENT FUNCTION	MISC. INFO
SINE & COSINE	2π	It takes one whole revolution of the unit circle before the outputs begin repeating themselves
TANGENT	π	Notice that this is not the same as sine and cosine. For tangent, the lower half of the unit circle repeats the outputs of the upper half.
COSECANT & SECANT	2π	Remember cosecant and secant are reciprocals of sine and cosine, so the period will be the same as sine and cosine.
COTANGENT	π	Cotangent is the reciprocal of tangent, so the period will be the same as tangent.

Notice that the graph of **y = sin(x)** shown below repeats every 2 pi.



Notice that the graph of **y = tan(x)** shown below repeats every pi.



Period Changes

Now that you have an idea of what the period is for the parent functions, what happens to the graph if you change the period?

Watch the video below to see how the period changing can affect the graph.

Video: How to Graph the Sine Graph with Period Change

Transformations of Trigonometric Graphs (Period Changes)

Below you will find the general equation for the sine function. Different textbooks use different variables, but as you can see, both equations are in the same format. For the purposes of this lesson, we will use the top version of the equation and focus on the **b** value.

$$y = a\sin(bx - c) + d$$

OR

$$y = A\sin(\omega x - h) + k$$

Period Changes

	Period
Sine Cosine Cosecant Secant	$Period = \frac{2\pi}{b}$
Tangent Cotangent	$Period = \frac{\pi}{b}$

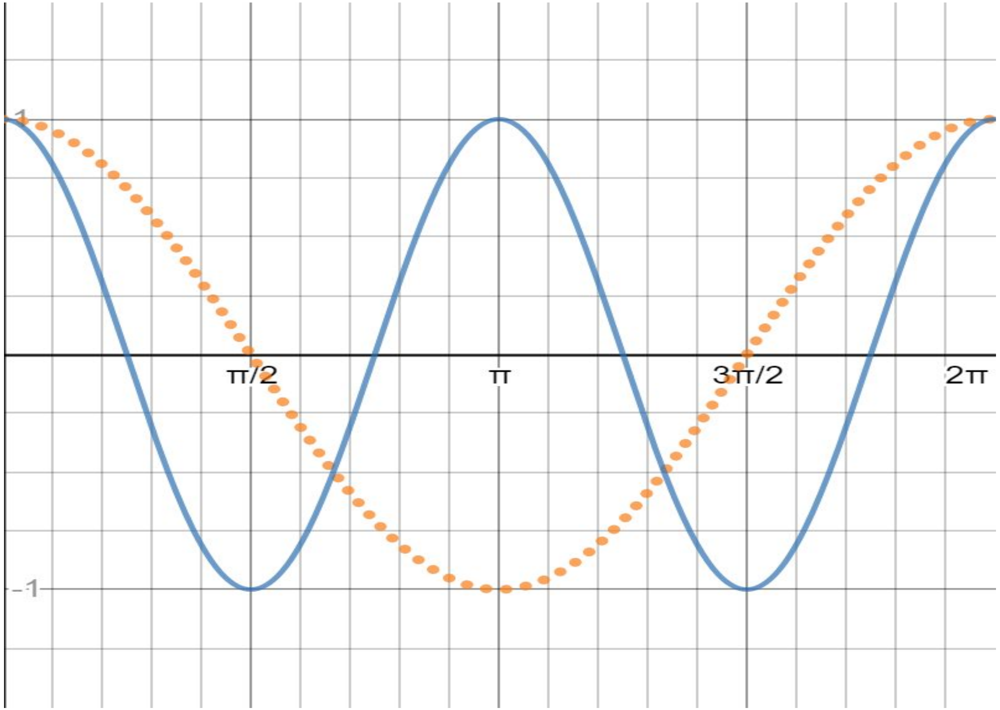
Example #1:

$$y = \cos(2x)$$

$$y = \cos(2x)$$

b = 2

$$Period = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$



Notice that the blue graph (**y = cos 2x**) completes the cycle twice in the time it takes the orange graph (**y = cos x**) to complete 1 cycle.

	Period
Sine Cosine Cosecant Secant	$Period = \frac{2\pi}{b}$
Tangent Cotangent	$Period = \frac{\pi}{b}$

Period Changes

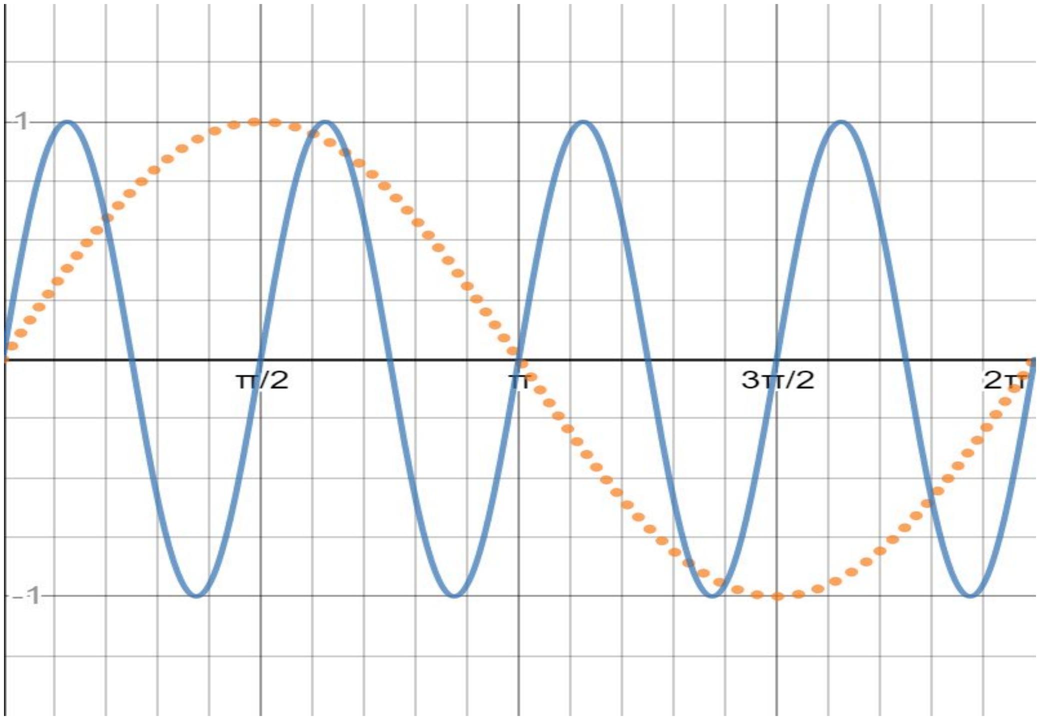
Example #2:

$$y = \sin(4x)$$

$$y = \sin(4x)$$

b = 4

$$Period = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$$



Notice that the blue graph (**y = sin 4x**) completes the cycle four times in the time it takes the orange graph (**y = sin x**) to complete 1 cycle.

Period Changes

	Period
Sine Cosine Cosecant Secant	$Period = \dfrac{2\pi}{b}$
Tangent Cotangent	$Period = \dfrac{\pi}{b}$

Example #3:

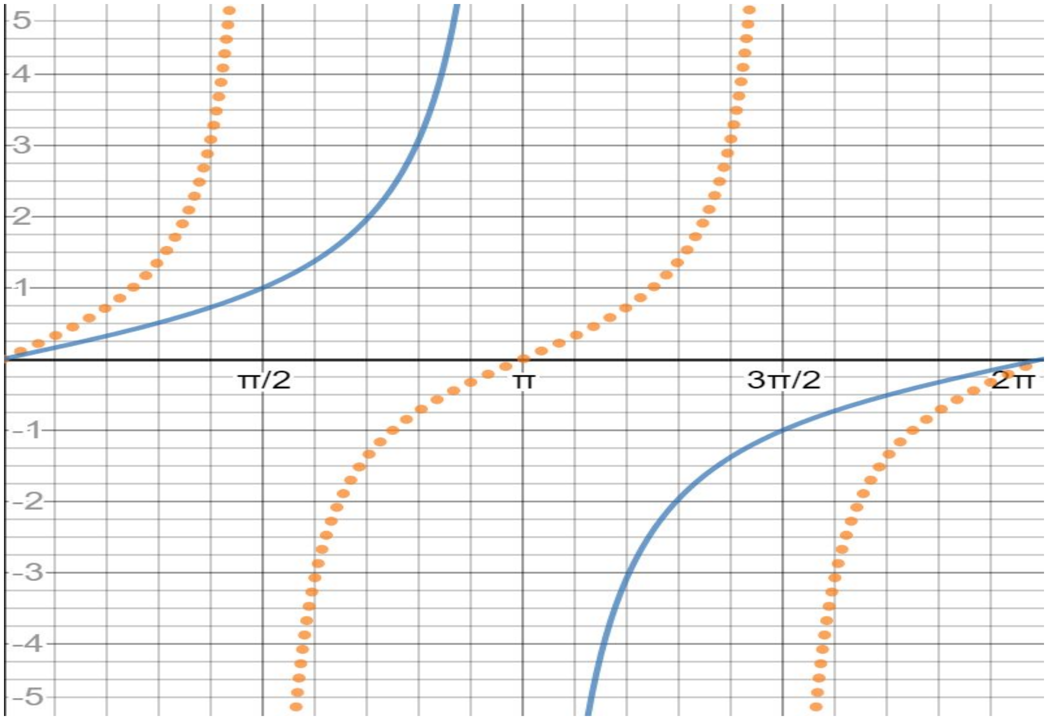
$$y = \tan\left(\frac{x}{2}\right)$$

$$y = \tan\left(\frac{x}{2}\right)$$

$$b = \frac{1}{2}$$

Notice that the numerator is pi, not 2pi for tangent.

$$Period = \frac{\pi}{b} = \frac{\pi}{\frac{1}{2}} = \frac{2}{1} \cdot \pi = 2\pi$$



Notice that the blue graph (**y = tan x/2**) takes twice as long to complete the cycle compared with the time it takes the orange graph (**y = tan x**) to complete 1 cycle. The orange graph starts repeating at pi.

Some things you might have noticed:

- As b gets bigger, the graph repeats more often. In other words, it gets skinnier.

- As b gets smaller, the graph takes longer before repeating. In other words it gets wider.

- b only affects the width, it does not affect the height.

- When determining the period for tangent or cotangent, don't forget to divide pi by b, NOT 2pi.

Graphing Period Changes Practice:

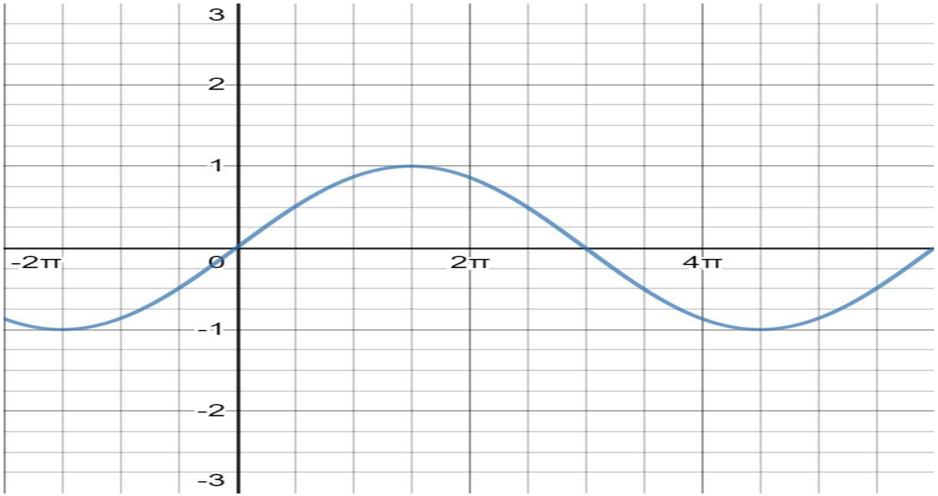
On a sheet of paper, sketch the graph. Then determine the domain and range for each.

1. $y = \sin(x/3)$
2. $y = \cos(4x)$
3. $y = \tan(2x)$
4. $y = \csc(x/2)$

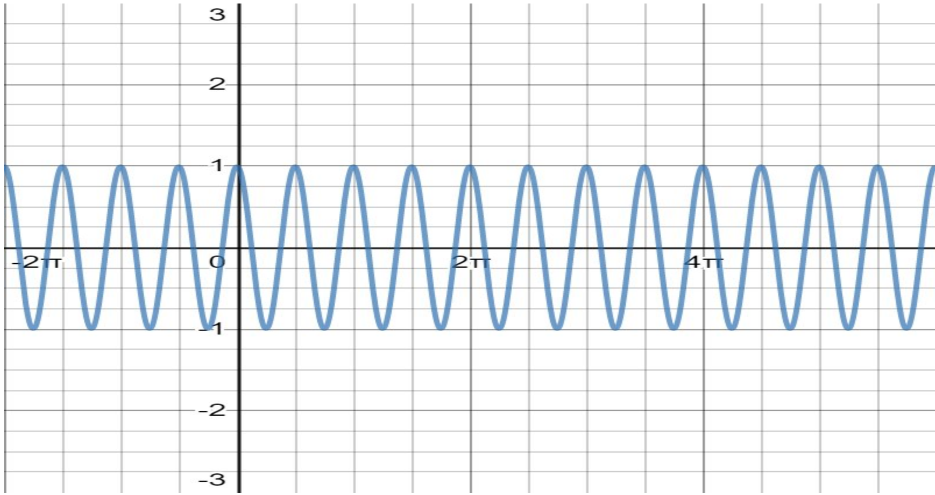
Graphing Period Changes Practice ANSWERS:

On a sheet of paper, sketch the graph. Then determine the domain and range for each.

1. y = sin(x/3)



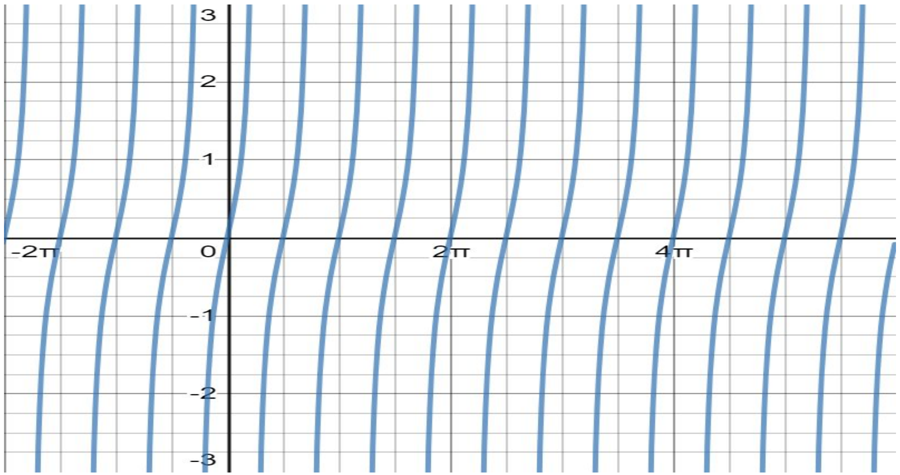
2. y = cos(4x)



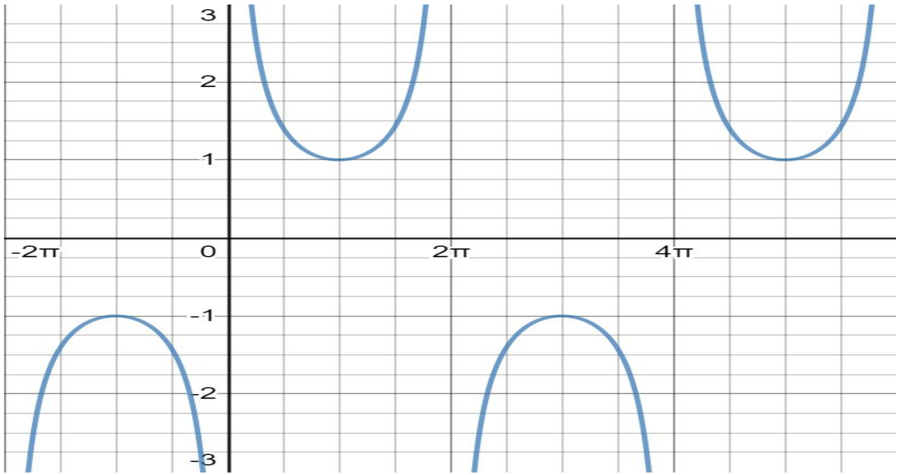
Graphing Period Changes Practice ANSWERS:

On a sheet of paper, sketch the graph. Then determine the domain and range for each.

3. $y = \tan(2x)$



4. $y = \csc(x/2)$



Graphing Period Changes Practice Answers:

On a sheet of paper, sketch the graph. Then determine the domain and range for each.

1. $y = \sin(x/3)$

Domain: All Real Numbers

Range: $\{\, y \mid -1 \leq y \leq 1 \,\}$ Or $[-1,1]$

2. $y = \cos(4x)$

Domain: All Real Numbers

Range: $\{\, y \mid -1 \leq y \leq 1 \,\}$ Or $[-1,1]$

3. $y = \tan(2x)$

Domain: $\left\{ x \,\middle|\, x \neq \frac{\pi}{4} + \frac{\pi}{2} \cdot n,\ where\ n\ is\ any\ integer \right\}$

Range: All Real Numbers

4. $y = \csc(x/2)$

Domain: $\{\, x \mid x \neq 2\pi \cdot n,\ where\ n\ is\ any\ integer \,\}$

Range: All Real Numbers

Additional Resources:

Click on the links below to get additional helpful videos as well as additional practice to check your understanding.

Additional Practice

[Graphing Trig Functions (with answers)]

[Graphs of Sine and Cosine Functions]

Helpful Videos

[Graphs of Sine and Cosine with Changes in Period, Amplitude, and Reflection]

[Transformations of Trigonometric Graphs: Amplitude, Period & Phase Shift]

(Watch **4:52 - 11:12** for Period Change or watch it all for overall help graphing)

[Graphing sine and cosine with period change]